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Abstract

In this paper, we report a puzzling result about the monetary expressions of labor time (MELTs) of the productive and unproductive sectors. Since part of the aggregate value produced in productive sectors is transferred to unproductive sectors, the productive sector's MELT is a measure of value realized in productive sectors while the unproductive sector's MELT is a measure of value transferred to unproductive sectors. Using the national income data for the U.S. economy during 1987-2016 and for the Korean economy during 1993-2016, it is found that the MELT of the aggregate productive sector and the MELT of the aggregate unproductive sector have been moving in a very close lockstep in both countries during the entire sample periods. We build a model which explicitly formalizes the unproductive sector as not producing any value but making the value production process efficient, and find that the co–movement of the two MELTs is not an optimal condition. We also suggest some *ex post* implications of it, including what the puzzling result implies on the relation between unproductive sector and capital accumulation.

Keywords: unproductive labor, monetary expression of labor time

JEL Classication: B51, E11, D46

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1 Introduction

One of the contributions of the New Interpretation (NI) of Marxian labor theory of value is to open up new research areas of empirical analyses of Marxian theory. Here, the monetary expression of labor time (MELT) plays a central role. Based on the idea that money value added results from expenditure of living labor, the NI proposes to adopt the equivalence between the two as an axiom, and the associated proportionality coefficient is the MELT, defined as aggregate money value added divided by total living labor time. Accordingly, as long as certain assumptions are made regarding various differences among sectors, the MELT can be a very useful tool to recover labor value variables from price variables simply by using national account data.

In Foley (1986), the MELT is defined at the aggregate level only, with unproductive labors excluded from the denominator. On the other hand, Rieu and Park (2018) extended the NI framework to estimate the MELTs at the industry level with an explicit distinction between productive versus unproductive industries. Note that in Marxian theory it is only productive industries that produce value, while part of the value thus–produced is transferred to unproductive industries. Accordingly, the MELT of the aggregate unproductive sector is defined as the ratio of the total money value transferred to the unproductive sector against total unproductive labor time, while the MELT of the aggregate productive sector is defined as the ratio of the total money value remained and realized within the productive sector against total productive labor time.

In this paper, using Rieu and Park (2018)'s approach, we estimated the MELT of the aggregate productive sector and that of the aggregate unproductive sector of Korea for 1993–2016 and of U.S. for 1987-2016. It was found, surprisingly, that the two time series of the MELT moved quite closely for the entire sample periods in both economies. This puzzling observation indicates that the volume of value transfer from the productive sector to the unproductive sector — however it is determined — has been so as to make the MELT of each sector evolve closely to each other.[1] This leads us to ask if there is any equilibrium or

[1] In the next section where the result is reported, we explain why it is a puzzle from the perspective of

optimal level of value transfer between the productive and unproductive sectors, or if our surprising finding is merely a coincidence.

To be more concrete, if the unproductive sector does not produce any value but only spends the redistributed values — whether as consumption or investment — a question might be raised at a fundamental level on why it exists and what its use is from the perspective of the economic system as a whole, or using Marxian terminology, from the perspective of total capital. But from the fact that the unproductive sector, which includes among others wholesale and retail trades, real estate, and finance, etc., has been an essential part of any capitalist economy, it follows that it must be the case that the unproductive sector is socially necessary although it does not create any value, hence justifying the value transfer for its maintenance. Then the next question is what an optimal size of the unproductive sector would be, proxied by the value transfer to the latter and the unproductive labor time.

As a way to address these questions related to the unproductive sector from the perspective of the total capital, we build a model to derive the optimal levels of the value transfer and the unproductive labor time; 'optimal' in the sense that they maximize the total profit of the productive sector, which is assumed as the source of capital accumulation and growth. One of the contributions of our model is to provide a formal definition of the unproductive sector being socially necessary and to explicitly incorporate it into the model. More specifically, in our model, while the unproductive industries do not create any value, they are socially necessary in the sense that they make the processes of creating value and surplus value efficient. This contradictory nature of the unproductive sector plays a key role in the model in deriving the optimal levels of the value transfer and the unproductive labor time.

The solution of the model demonstrates that the optimal levels of the MELTs of the productive sector and the unproductive sector are not equal to each other, from which we conclude that the two MELTs moving in close lockstep may not be an optimal condition. Then, instead of further pursuing to provide a possible explanation of the puzzle, we take an indirect route of using the model to identify its *ex post* implications. First, we conduct comparative statics analysis to see how the optimal levels of the value transfer and unpro-

Marx's transformation procedure.

ductive labor time respond to a change in parameters of the model in an economy where our puzzling empirical finding is observed. The results of the comparative statics analysis shed light on the implication of the co–movement of the two MELTs on the distribution of value between the productive and unproductive sectors.

Second, we take a more specific issue of how the contradictory nature of unproductive sector relates to capital accumulation and growth. Since the productive sector does not produce values, it is common in the literature to view it as undermining the growth potential of the system. For instance, using a two–sector model Dutt (1991) provides a Marxian explanation of stagnation caused by the growth of unproductive sector. By contrast, we explore the possibility of capital accumulation and growth led by unproductive sector under a certain condition.[2] For this, we suggest the concept of unproductive sector–led phase, defined as a phase where the economy can rely on expanding the unproductive sector for accumulation and growth. In the important analytical result emerging from this framework, it matters whether the economy is faced by labor supply constraint or not. On one hand, the economy with unconstrained labor supply is unambiguously unproductive sector–led regardless of whether the co–movement of the MELTs of the productive and unproductive sectors is observed or not. This result comes out as obvious mathematically, via the envelope theorem.

On the other hand, when the economy is constrained by labor supply, it depends on the wage rate in the productive sector; if the productive sector wage rate is high relative to the conditions which affect labor productivity in that sector, the economy can rely on the expansion of unproductive sector for capital accumulation and growth. That is, if the conditions that affect the productive sector labor productivity in the economy constrained by labor supply are sufficiently disadvantageous, the economy will be able to achieve accumulation and growth by expanding the unproductive sector; but if the labor productivity conditions in the productive sector are sufficiently favorable, the economy will achieve accumulation and growth by shrinking the unproductive sector.

However, when the co–movement of the MELTs of the productive and unproductive sectors is observed in the economy constrained by labor supply, the conditions that affect the

[2]As will be seen below, the condition has to do with the wage rate in the productive sector.

productive sector labor productivity become unimportant in deciding whether the economy is unproductive sector–led or not. The fact that the co–movement has been observed in the economy constrained by labor supply implies that the accumulation property of the economy in terms of productive versus unproductive sectors is not affected by the labor productivity conditions of the productive sector.

The rest of the paper is organized as follows. We start with reporting the puzzling result of the MELTs of the productive and unproductive sectors in Korea and U.S. in section 2. Section 3 introduces the basic setup of the model. In section 4, an economy with unconstrained labor supply is examined as the benchmark; we derive the optimal solution of the model and suggest the concept of unproductive sector–led phase. We then move on to an economy constrained by labor supply in section 5, derive the optimal solution, and examine whether the economy is unproductive sector–led. Section 6 is conclusion.

2 A puzzle about the MELT

Let us first introduce notation. MVA denotes aggregate money value added and L denotes total living labor time. In order to have the productive–unproductive distinction, we use subscript P and U to indicate productive sector and unproductive sector, respectively.

According to the productive–unproductive distinction, it is only productive labor that produces value and surplus value. Therefore, value and surplus value are produced only within productive sectors, while unproductive sectors are fed by value transferred from the productive sectors. Then we have

$$MVA_P = MVA - MVA_U \tag{1}$$

which is the value remained and realized within the productive sectors after the value transfer to the unproductive sectors.

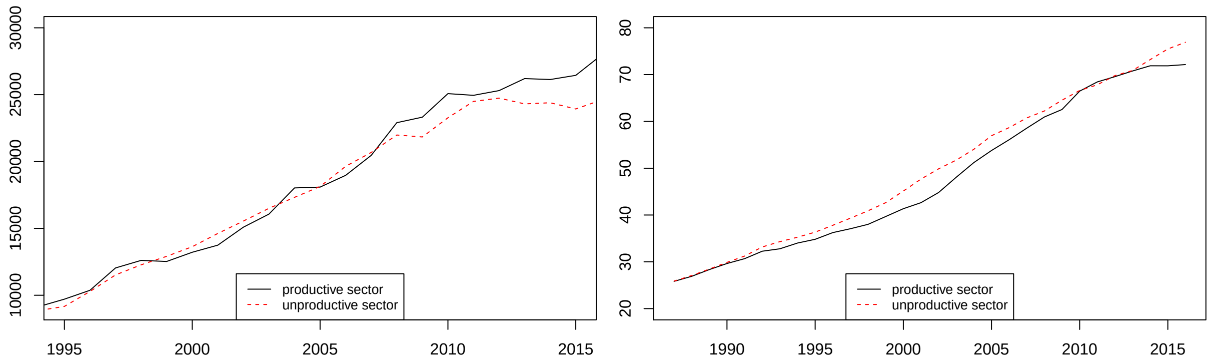
In this framework, three distinctive measures of the MELT can be defined as follows.

(i) $m = \frac{MVA}{L_P}$: value produced per productive labor time

(ii) $m_P = \frac{MVA_P}{L_P}$: value realized, after value transfer, per productive labor time

Figure 1: The MELTs of productive vs. unproductive sectors

(a) Korea (1993–2016; KRW per hour)   **(b)** U.S. (1987–2016; USD per hour)



(iii) $m_U = \frac{MVA_U}{L_U}$: value transfer per unproductive labor time

Under the assumption that all labors in productive sectors are productive while all labors in unproductive sectors are unproductive — which is the assumption we adopt in this paper[3] — m, the aggregate MELT, is a measure of aggregate value produced in the whole economy by a given total productive labor time; m_P, the productive sector MELT, is a measure of total value realized within the productive sectors, after the value transfer, for a given total productive labor time; m_U, the unproductive sector MELT, is a measure of total value transferred to unproductive sectors for a given total unproductive labor time. The relative levels of m_P and m_U should be a matter of critical importance since in Marxian theory the surplus value is the source of capital accumulation and growth of the system.

In this context, using the national account data of Korea for 1993–2016 and the national account data of U.S. for 1987–2016, we estimated m_P and m_U to gauge the value transfer between productive and unproductive sectors.[4] The results are reported in figure 1. It is

[3] Mohun (2006) separates between productive and unproductive labors within productive sectors.

[4] The productive industries include agriculture, forestry, and fishery; mining; manufacturing; utilities; construction; accommodation and food service; transportation and warehousing; information services; education service; arts and entertainment. The unproductive industries include wholesale/retail trade; finance, insurance, real estate, rental and leasing; professional and business service; health care and social services; other services except government.

surprising that, during the protracted periods of time, both Korea and U.S. have seen the MELT of productive sector and the MELT of unproductive sector closely tracking each other to the extent that it almost holds that

$$m_P = m_U \tag{2}$$

Is this relation a consequence of some equilibrating mechanism or just a coincidence? Of course, extending the data to see if the same puzzling relation would be observed in other countries will get us closer to an answer to this question. But the observation of $m_P = m_U$ for both Korea and U.S. during the long sample period is puzzling enough to bring us to investigate an underlying mechanism or economic implication. Whatever it may be, the observation emphatically demonstrates that the volume of total value transferred to unproductive sectors from productive sectors has been such that $m_P = m_U$ holds. Then does this imply that there is some 'optimal' level of value transfer at the aggregate level from the perspective of long run stability of the macroeconomy?

That $m_P = m_U$ is a puzzle is obvious from the perspective of Marxian transformation procedure. Suppose an economy aggregated into productive and unproductive sectors, which yields a two–sector model. Unless the organic composition of capital of each sector is equal to one another, which takes place only by chance, the ratio between the two sectors' money value added and that between their value products will be different; hence $m_P \neq m_U$. Although not reported here, we estimated the organic composition of capital, proxied by a capital stock divided by total wage, of productive and unproductive sectors in Korea during 1993–2016 and verified an evident difference between the two sectors.[5]

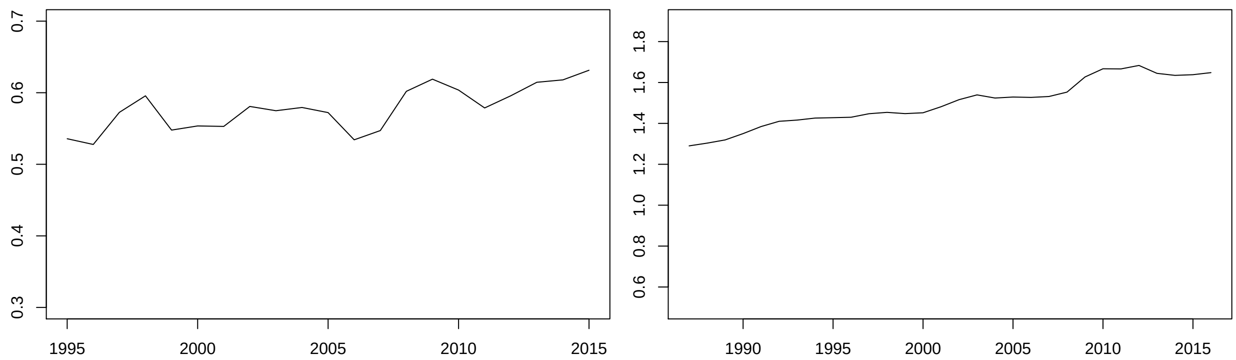
In the rest of this paper, we propose an analytical approach that can be useful to address some of the above–mentioned questions surrounding the puzzling result of (2).

Before proceeding, the unproductive sector's total labor time relative to productive sector's total labor time, $\frac{L_U}{L_P}$, is illustrated in figure 2. An expansion of unproductive sectors in terms of total work hours is clearly observed in both countries during the entire sample periods.

[5] The data can be provided upon request.

Figure 2: The unproductive–productive sectors ratio of total labor time ($\frac{L_U}{L_P}$)

(a) Korea (1995–2015) **(b)** U.S. (1987–2016)



3 A model: basic setup

In this section, we introduce the basic setup of a model to investigate the implications of unproductive sector and its expansion in relation to value and surplus value production, capital accumulation, and growth.

First of all, the total value produced by productive labors for a given m is

$$MVA = mL_P \tag{3}$$

On the other hand, the profit realized in the productive sectors, which we suppose to be the only source for capital accumulation and growth of the system, is expressed as, by definition, $\Pi_P = MVA_P - w_P L_P$. Then, using equations (3), (1), and $MVA_U = m_U L_U$, we get an expression for the productive sector profits.

$$\Pi_P = mL_P - m_U L_U - w_P L_P \tag{4}$$

Equation (4) schematically reflects the Marxian concept of productive vs. unproductive labors; whether productive or unproductive, all types of labor incur cost and hence are a pressure on capital profitability — see the second and third terms on the right–hand side; on the other hand, the productive labors produce value and surplus value — see the first term on the right–had side — while this is not the case for the unproductive labors; the latter simply subtract from value and surplus value produced by the productive labors.

Note that the total value transfer to the unproductive sectors from the productive sectors, measured by $MVA_U = m_U L_U$, has two components, i.e. the unproductive labor time L_U and the value transfer per unit unproductive labor time m_U. An increase in L_U with other things being equal can be viewed as a quantitative expansion of the unproductive sector, whereas an increase in m_U for given L_U may reflect an enhancement in technological and market condition of the unproductive sectors and hence can be viewed as a qualitative expansion of the unproductive sector.

Whether qualitative or quantitative, however, equation (4) emphasizes that an expansion of the unproductive sector is a drain on the source of capital accumulation, i.e. profits, and thus lowers the growth of the system. However, it is recognized in the literature that there are some aspects of unproductive labors which make them 'socially necessary'. For instance, Smith (1993) maintains that some unproductive labors are 'socially necessary' and hence should be considered as overhead costs. On the other hand, Olsen (2015) suggests that unproductive labors, while squeezing profits and hence lowering growth, also exert some countering effect through increasing work intensity and developing productivity enhancing technical change.

In this context, we propose to characterize the unproductive sector — measured not only by unproductive labor but also by the value transfer — as 'socially necessary' in the sense that it makes the process of production of value and surplus value efficient.[6] To express this formally, let m be a function of m_U and L_U in addition to other variables that positively affect m which we denote by a comprehensive variable α.

$$m = m(m_U, L_U, \alpha) \tag{5}$$

where $\frac{\partial m}{\partial \alpha} > 0$ holds. α captures technological and market conditions of the productive sector, which enhance the value creating capacity of any given productive labor time.[7]

As for the impact of m_U and L_U on m, we adopt the following assumptions.

[6] We are using 'efficient' in the usual sense of the word as a nuanced expression only for the convenience.

[7] Remember that the technological and market conditions of the unproductive sector are reflected in the determination of m_U.

Assumption 1 $\frac{\partial m}{\partial m_U} > 0$*: For given* L_U*, an increase (a decrease) in the value transferred to unproductive sectors makes the value creation by productive labors more (less) efficient.*

Assumption 2 $\frac{\partial m}{\partial L_U} > 0$*: For given* m_U*, i.e. when the value transferred to unproductive sectors per unproductive labor time is constant, an increase (a decrease) in unproductive labor time makes the value creation per productive labor time more (less) efficient.*

Under these two assumptions, both qualitative and quantitative expansions of the unproductive sector make the value production by the productive labor efficient; the unproductive sector contributes to the value production in this sense. That is, while the productive labor directly contributes to the value production, the unproductive labor's contribution is only indirect. It is in this way that we characterize the unproductive sector as socially necessary and adopt the following definition.

Definition 1 *Unproductive sectors are socially necessasry insofar as assumptions 1 and 2 hold; i.e.* $\frac{\partial m}{\partial m_U} > 0$ *and* $\frac{\partial m}{\partial L_U} > 0$*. The levels of* $\frac{\partial m}{\partial m_U}$ *and* $\frac{\partial m}{\partial L_U}$ *are a measure of the degree by which the value transfer and the unproductive labors, respectively, make the value production per productive labor efficient.*

Now, recalling the questions raised in section 1 regarding the relation of $m_P = m_U$, in order to see whether there are some kind of 'optimal' magnitudes of unproductive sector and value transfer and how they are determined, we choose m_U and L_U as two key unknowns with w_P and L_P taken as given. Once m_U and L_U are found, m and Π_P will be known from equations (5) and (4), and finally we will be able to get m_P from its definition:

$$m_P = \frac{mL_P - m_U L_U}{L_P} \tag{6}$$

In this context, let us suppose the existence of what Engels (1970) calls the ideal personification of the total national capital (ideeller Gesamtkapitalist), a hypothetical agent who decides the level of m_U and L_U that maximize profit of total capital. Of course a decentralized market economy doesn't have such a social optimizer. We use the optimization of the total capital as a counter–factual method to examine what type of relation between productive and unproductive sectors would be desirable from the bird's eye view of the total capital and investigate the implication.

In analyzing this optimization problem, we consider two cases in turn; an economy with unconstrained labor supply and an economy constrained by labor supply. When there is labor supply constraint, a quantitative expansion of unproductive sector, i.e. an increase in unproductive labor time, diminishes the pool of labor for productive employment, while this is not the case when there is no such constraint. Underdeveloped and developing countries with huge non–capitalist sectors, which may be the source of elastic labor supply to industrialized cities, are an example of an economy with unconstrained labor supply. On the other hand, developed countries where such non–capitalist sectors are very small are likely to be an economy constrained by labor supply, although the constraint can possibly be relaxed by, for example, immigration, female participation in labor force, etc.

4 An economy with unconstrained labor supply

We first consider an economy with unconstrained labor supply.

4.1 Optimal value transfer

The optimization problem of 'ideeller Gesamtkapitalist' is as follows.

$$\max_{m_U, L_U} \quad \Pi_P = m(m_U, L_U, \alpha)L_P - m_U L_U - w_P L_P$$

The solution is summarized in lemma 1.

Lemma 1 *The optimal solution of the model economy with unconstrained labor supply:*

$$L_U^* = \frac{\partial m}{\partial m_U} L_P \tag{7}$$

$$m_U^* = \frac{\partial m}{\partial L_U} L_P \tag{8}$$

$$m_P^* = m^* - \frac{\partial m}{\partial m_U}\frac{\partial m}{\partial L_U} L_P \tag{9}$$

$$m^* = m(L_U^*, m_U^*, \alpha) \tag{10}$$

$$\Pi_P^* = m^* L_P - \frac{\partial m}{\partial m_U}\frac{\partial m}{\partial L_U} L_P^2 - w_P L_P \tag{11}$$

Proof. From the first order conditions

$$\frac{\partial \Pi_P}{\partial m_U} = \frac{\partial m}{\partial m_U} L_P - L_U = 0$$
$$\frac{\partial \Pi_P}{\partial L_U} = \frac{\partial m}{\partial L_U} L_P - m_U = 0$$

we obtain L_U^* and m_U^*. The rest can be shown by substituting m^*, m_U^*, and L_U^* to equations (4), (5), and (6). ■

The idea behind the solution of the two choice variables is clear and intuitive. As the unproductive sectors make the value creation of productive labor more (less) efficient, the number of unproductive labor, L_U, and the value transfer to the unproductive sectors, m_U, for a given L_P will be great (smaller). Probably the rising trends of $\frac{L_U}{L_P}$ in Korea and U.S. as displayed in figure 2 reflect an increase in $\frac{\partial m}{\partial m_U}$ due to technological innovations, that took place in unproductive sectors during last decades.[8] The levels of m_U and L_U beyond (8) and (7) are suboptimal, crowding out capital accumulation and growth, and will be considered as 'too big'. Some of the findings that financial sector crowds out real economic growth, such as Cecchetti and Kharroubi (2015), may be understood in this context.

Corollary 1, which presents an interesting result about an economy without an unproductive sector, directly follows from the optimal solution in lemma 1.

Corollary 1 *In an economy with unconstrained labor supply, if* $\frac{\partial m}{\partial m_U} = 0$ *and* $\frac{\partial m}{\partial L_U} = 0$*, then (i)* $L_U^* = 0$ *and* $m_U^* = 0$*, and (ii)* $\Pi_P^* \gtreqless 0$ *as long as* $m_P^* \gtreqless w_P$*.*

Proof. On one hand, (i) is immediate from (7) and (8). On the other hand, it is confirmed from (9) that $m_P^* = m^*$ and from (11), due to $m_P^* = m^*$, that $\Pi_P^* = (m_P^* - w_P)L_P$; (ii) is immediate from the latter. ■

Part (i) of corollary 1 implies that if the unproductive sectors were not socially necessary in the sense of definition 1, the unproductive labors would not exist and consequently there

[8] Think about innovations that took place in financial intermediation and retail trade (e.g. Amazon.com), both of which being the two major unproductive sectors.

would be no value transfer to the unproductive sectors; part (ii) implies, nonetheless, profit will still be produced as long as the wage rate is not so high as to eliminate profit.[9]

The most important result from the optimal solution in lemma 1 is to verify $m_P^* \neq m_U^*$, which tells us that the relation $m_P = m_U$ observed in Korea and U.S. is not established as an optimal condition. Then, instead of further pursuing to identify an underlying mechanism of $m_P = m_U$, we now turn to clarifying its *ex post* implication. Let us start with expressing the definition $MVA = MVA_P + MVA_U$ at the optimum as

$$m^* L_P = m_P^* L_P + m_U^* L_U^* \tag{12}$$

Imposing $m_U^* = m_P^*$ gives

$$\widetilde{m}^* L_P = \widetilde{m}_U^* L_P + \widetilde{m}_U^* \widetilde{L}_U^* = \widetilde{m}_U^* (L_P + \widetilde{L}_U^*) \tag{13}$$

Throughout the paper, the variables under the condition of $m_U = m_P$ are denoted with tilde.

Meanwhile, according to the definition of the optimal total profit of productive sectors under the condition of $m_U^* = m_P^*$, we get

$$\widetilde{m}^*(\widetilde{m}_U^*, \widetilde{L}_U^*, \alpha) L_P - \widetilde{m}_U^* \widetilde{L}_U^* - w_P L_P = \widetilde{m}_U^* L_P - w_P L_P \tag{14}$$

The left–hand side is the value function of ideeller Gesamtkapitalist's optimization problem and the right–hand side is obtained from the definition of the optimal total profit of productive sectors, $m_P^* L_P - w_P L_P$ with $m_U^* = m_P^*$ extrapolated.

Now, using the envelope theorem, we obtain some interesting comparative statics results with respect to the parameters α and L_P.[10]

Proposition 1 *In an economy with unconstrained labor supply where $m_P^* = m_U^*$ holds, it follows that*

$$\frac{\partial m}{\partial \alpha} = \frac{\partial \widetilde{m}_U^*}{\partial \alpha} \tag{15}$$

[9]For an economy constrained by labor supply discussed below, we obtain a stronger result where the wage rate is always set at the level that eliminates all the profit.

[10]The comparative statics with respect to another parameter of the model, w_P, generates a trivial result.

Proof. Taking the derivative of both sides of (14) w.r.t α yields $\frac{\partial m}{\partial \alpha}L_P$ for the left–hand side, where the enveloped theorem is used, and $\frac{\partial \widetilde{m}_U^*}{\partial \alpha}L_P$ for the right–hand side; we eventually obtain $\frac{\partial m}{\partial \alpha} = \frac{\partial \widetilde{m}_U^*}{\partial \alpha}$. ∎

Proposition 1 states that when m rises (falls) due to an improvement (worsening) of the technological and market conditions of the productive sector, m_U also rises (falls) by the same degree. This means that since $m_P = m_U$, a variation of the technological and market conditions of the productive sector will make all the MELTs, m, m_P, and m_U, change by the exact same amount. This result gauges the way in which the additional value production, generated by technological innovations and market condition improvement of the productive sector, is shared between the productive and unproductive sectors.

Proposition 2 *In an economy with unconstrained labor supply where $m_P^* = m_U^*$ holds, it follows that*

$$\frac{\widetilde{L}_U^*}{L_P} = \frac{\partial \widetilde{m}_U^*}{\partial L_P}\frac{L_P}{\widetilde{m}_U^*} \tag{16}$$

Proof. Taking the derivative of both sides of (14) w.r.t L_P yields $m - w_P$ for the left–hand side, where the envelope theorem is used, and $\frac{\partial \widetilde{m}_U^*}{\partial L_P}L_P + \widetilde{m}_U^* - w_P$ for the right–hand side; we eventually obtain

$$m - \widetilde{m}_U^* = \frac{\partial \widetilde{m}_U^*}{\partial L_P}L_P \tag{17}$$

Dividing both sides by $\widetilde{m}_U^*$ to make the right–hand side an elasticity,

$$\frac{m - \widetilde{m}_U^*}{\widetilde{m}_U^*} = \frac{\partial \widetilde{m}_U^*}{\partial L_P}\frac{L_P}{\widetilde{m}_U^*} \tag{18}$$

Using (13), the above expression can be rewritten as $\frac{\widetilde{L}_U^*}{L_P} = \frac{\partial \widetilde{m}_U^*}{\partial L_P}\frac{L_P}{\widetilde{m}_U^*}$. ∎

According to proposition 2, the elasticity of $\widetilde{m}_U^*$ in response to L_P equals the ratio between unproductive and productive labor time. That is, as the relative importance of unproductive labor time in total employed population gets heightened, the total money value added, which has risen due to an increase in productive labor time, tends to be more easily transferred to unproductive sectors. This result motivates a political interpretation of the dynamic between productive and unproductive labors surrounding the distribution of surplus value.

4.2 Unproductive sector and capital accumulation

To be more concrete about the *ex post* implication of $m_P = m_U$, we tackle a critical issue on how unproductive sectors relate to capital accumulation and growth. For this, let us hypothesize an economy in a phase where capital accumulation and growth are led by unproductive sectors. To capture this idea, we introduce the concept of 'unproductive sector–led phase'; for this, m in (5) needs to be specified as a linear function such as

$$m = \beta_1 m_U + \beta_2 L_U + \beta_3 \alpha \qquad \beta_1, \beta_2, \beta_3 > 0 \tag{19}$$

Note that $\beta_1 \equiv \frac{\partial m}{\partial m_U}$ and $\beta_2 \equiv \frac{\partial m}{\partial L_U}$ are the two measures of the degree by which unproductive sectors — measured by either m_U or L_U — make the production of value efficient as discussed in definition 1. Since an increase in L_U is a quantitative expansion of the unproductive sector while an increase in m_U for give L_U is a qualitative expansion of the unproductive sector, β_1 can be called the qualitative effect of unproductive sector efficiency and β_2 the quantitative effect of unproductive sector efficiency. Similarly, β_3 measures the efficiency of productive sector's technological and market conditions in value production and hence can be called the qualitative effect of productive sector efficiency.

These efficiency coefficients, β_1, β_2, and β_3, are the deep parameters of the model that measure more structural aspects of the contributions m_U, L_U, and α make to increasing m. In the definition of the unproductive sector–led phase suggested in this paper, we focus on how the two efficiency coefficients of the unproductive sector, β_1 and β_2, affect the productive sector's profits, which are the source of accumulation and growth.

Definition 2 *Suppose a linear specification of m as in* (19)*. An economy is said to be unproductive sector–led if both of the following two conditions hold simultaneously:*

$$\frac{\partial \Pi_P^*}{\partial \beta_1} \geqslant 0 \quad \textit{and} \quad \frac{\partial \Pi_P^*}{\partial \beta_2} \geqslant 0$$

with at least one of them holding with strong inequality.

According to definition 2, an economy is said to be unproductive sector–led when an improvement of either the qualitative or the quantitative effect of unproductive sector efficiency raises the optimal profit of productive sector while neither lowers it.

Now, before proceeding to examine whether an economy where $m_P = m_U$ holds is unproductive sector–led, we first consider the general case without $m_P = m_U$ as a reference point; the result is summarized in proposition 3.

Proposition 3 *According to definition 2, the economy with unconstrained labor supply is unproductive sector–led since*

$$\frac{\partial \Pi_P^*}{\partial \beta_1} = \beta_2 L_P^2 > 0$$
$$\frac{\partial \Pi_P^*}{\partial \beta_2} = \beta_1 L_P^2 > 0$$

Proof. Under the linear specification of m as in (19), Π_P^* in (11) is rewritten as

$$\Pi_P^* = \beta_0 L_P + \beta_1 \beta_2 L_P^2 + \beta_3 \alpha L_P - w_P L_P$$

The results in proposition 3 immediately follow. ■

What about the economy where $m_P = m_U$ holds? To check this, we have to first impose $m_P^* = m_U^*$ to the optimal solutions listed in lemma 1. The consequently revised solutions are listed in lemma 2.

Lemma 2 *In an economy with unconstrained labor supply, the optimal solution under the condition of* $m_P^* = m_U^*$*:*

$$\widetilde{L}_U^* = \beta_1 L_P \tag{20}$$
$$\widetilde{m}_U^* = \beta_2 L_P \tag{21}$$
$$\widetilde{m}_P^* = \beta_2 L_P \tag{22}$$
$$\widetilde{m}^* = \beta_2 (1 + \beta_1) L_P \tag{23}$$
$$\widetilde{\Pi}_P^* = \beta_2 L_P^2 - w_P L_P \tag{24}$$

Using equation (24), we can verify whether the economy in question is unproductive sector–led. The result is summarized in proposition 4.

Proposition 4 *According to definition 2, an economy unconstrained by labor supply where* $m_P^* = m_U^*$ *is observed is unproductive sector–led since*

$$\frac{\partial \widetilde{\Pi}_P^*}{\partial \beta_1} = 0$$
$$\frac{\partial \widetilde{\Pi}_P^*}{\partial \beta_2} = L_P^2 > 0$$

Proof. The results immediately follow from (24). ■

Propositions 3 and 4 combined together state that in the economy with unconstrained labor supply, regardless of whether $m_U = m_P$ holds or not, is unproductive sector–led in the sense that an improvement in the qualitative or the quantitative effect of the unproductive sector efficiency or both leads to an increase in the capital profitability. That is, the economy with unconstrained labor supply can always improve capital accumulation and growth by relying on the unproductive sectors' socially necessary character of making the production of value and surplus value efficient.

In fact, this result is obvious mathematically. Consider the productive sector profits in (4), which is the object function of the optimization problem. By using the envelope theorem, it can be readily confirmed that as long as m has a linear function as in (19), an increase in β_1 or in β_2 will unambiguously raise the profit; hence unproductive sector–led.

It can be concluded that in an economy with unconstrained labor supply the condition $m_P = m_U$ does not have any significant implication regarding the relation between capital accumulation and unproductive sector. But the matter is different when we turn to an economy constrained by labor supply as will be discussed in the next section.

5 An economy constrained by labor supply

For developed and mature economies such as South Korea and U.S., it is more realistic to assume labor supply elasticity to be quite small. In this section we consider an economy constrained by labor supply; in contrast to the economy dealt with in the previous section, now the total labor supply is fixed and as a consequence an increase in the unproductive labor time reduces the productive labor time.

5.1 Optimal value transfer

In the economy constrained by labor supply, the 'ideeller Gesamtkapitalist' solves the following constrained optimization problem.

$$\begin{aligned} \max_{m_U, L_U} \quad & \Pi_P = m(m_U, L_U, \alpha) L_P - m_U L_U - w_P L_P \\ \text{s.t.} \quad & \bar{L} = L_P + L_U \end{aligned}$$

which is simplified to

$$\max_{m_U, L_U} \quad \Pi_P = m(m_U, L_U, \alpha)(\bar{L} - L_U) - m_U L_U - w_P(\bar{L} - L_U)$$

The solution is summarized in lemma 3.

Lemma 3 *The optimal solution of the model economy constrained by labor supply:*

$$L_U^* = \frac{\frac{\partial m}{\partial m_U}}{1 + \frac{\partial m}{\partial m_U}} \bar{L} \tag{25}$$

$$m_U^* = \frac{\frac{\partial m}{\partial L_U}}{1 + \frac{\partial m}{\partial m_U}} \bar{L} + w_P - m^*(m_U^*, L_U^*, \alpha) \tag{26}$$

$$m^* = m(m_U^*, L_U^*, \alpha) \tag{27}$$

$$m_P^* = m^* \Big(1 + \frac{\partial m}{\partial m_U}\Big) - w_P \frac{\partial m}{\partial m_U} - \Big(\frac{\frac{\partial m}{\partial m_U} \frac{\partial m}{\partial L_U}}{1 + \frac{\partial m}{\partial m_U}}\Big) \bar{L} \tag{28}$$

$$\Pi_P^* = (m^* - w_P)\bar{L} - \frac{\frac{\partial m}{\partial m_U} \frac{\partial m}{\partial L_U}}{\Big(1 + \frac{\partial m}{\partial m_U}\Big)^2} \bar{L}^2 \tag{29}$$

Proof. From the first–order conditions

$$\begin{aligned} \frac{\partial \Pi_P}{\partial m_U} &= \frac{\partial m}{\partial m_U}(\bar{L} - L_U) - L_U = 0 \\ \frac{\partial \Pi_P}{\partial L_U} &= \frac{\partial m}{\partial L_U}(\bar{L} - L_U) - m - m_U + w_P = 0 \end{aligned}$$

we obtain m_U^* and L_U^*. The rest follows immediately. ■

The result for L_U^* in equation (25), on one hand, is identical to the one in (7) of the unconstrained optimization problem, i.e. higher $\frac{\partial m}{\partial m_U}$, higher L_U for a given L_P, corresponding to the observation illustrated in figure 2. On the other hand, m_U^* in equation (26) is not in a closed–form. Therefore, the existence and uniqueness of the solution m_U^* needs to be verified, which is done in lemma 4.

Lemma 4 *In the case of an economy constrained labor supply, the unique solution m_U^* exists.*

Proof. Let $g(x;\cdots) = \frac{\frac{\partial m}{\partial L_U}}{1+\frac{\partial m}{\partial m_U}}\bar{L}+w_P-m^*(x,L_U^*,\alpha)$, which is the right–hand side of equation (26) with m_U^* replaced by some variable x. Since g is continuous and monotonic, according to a fixed point theorem, there exists a fixed point $\bar{x}$ such that $g(\bar{x})=\bar{x}$, and we have $\bar{x}=m_U^*$. ■

As a comparison to corollary 1, here we consider an absence of unproductive sector in an economy constrained by labor supply. The result is summarized as follows.

Corollary 2 *In an economy constrained by labor supply, if $\frac{\partial m}{\partial m_U}=0$ and $\frac{\partial m}{\partial L_U}=0$, then not only (i) $m_U^*=0$ and $L_U^*=0$, but also, in comparison to part (ii) of corollary 1, (ii) $\Pi_P^*=0$.*

Proof. Suppose $\frac{\partial m}{\partial m_U}=0$ and $\frac{\partial m}{\partial L_U}=0$. On one hand, $L_U^*=0$ is immediate from (25). On the other hand, we obtain from (28) $m_P^*=m^*$ which, when substituted into the definition of m_P in (6), gives $\frac{m_U^*L_U^*}{\bar{L}-L_U^*}=0$. The latter, due to the definition $m_U^*\equiv\frac{MVA_U^*}{L_U^*}$, is rewritten as $\frac{MVA_U^*}{\bar{L}-L_U^*}=0$ or, since $L_U^*=0$, $\frac{MVA_U^*}{\bar{L}}=0$, which finally implies $MVA_U^*=0$ and thus $m_U^*=0$.[11] We also obtain from (26) $m_U^*=w_P-m^*$ which, due to $m_U^*=0$ and $m_P^*=m^*$, implies $w_P=m_P^*$. Finally, with the help of $m_P^*=m^*$, (29) is simplified to, $\Pi_P^*=(m_P^*-w_P)\bar{L}$, which, due to $w_P=m_P^*$, yields $\Pi_P^*=0$. ■

Corollary 2 suggests that in an economy constrained by labor supply, when the unproductive sectors are not socially necessary in the sense of definition 1, it is not only that the unproductive sectors will not exist but also that no profit will be produced. For a comparison, recall corollary 1 for the economy with unconstrained labor supply where the result of

[11] $m_U^*=0$ can be derived from a slightly different approach. According to the definition $m_U^*\equiv\frac{MVA_U^*}{L_U^*}$, it would be more precise to note that m_U^* cannot be defined since $L_U^*=0$. This can be interpreted as follows: $L_U^*=0$ is equivalent to a complete absence of the unproductive sector unless the latter is characterized by full automation technology; and the value transfer to a sector that does not exist would be a non sequitur. In this context, $m_U^*=0$ would be a reasonable take.

an absence of unproductive sector is $\Pi_P^* \gtreqless 0$ if and only if $m_P^* \gtreqless w_P$. According to corollary 2, however, in the economy constrained by labor supply, the result of an absence of unproductive sector is always $m_P^* = w_P$ and hence $\Pi_P^* = 0$.[12] Since $m_P = m$ measures the value production of a given productive labor time when there is no unproductive sector, corollary 2 suggests that when an economy constrained by labor supply doesn't have any unproductive sector, the wage rate tends to be at the level, which allows the workers to glean everything they produce, leaving nothing for capital.

Corollary 2 can be compared to the result of Okishio (2001)'s two–sector model, which shows that when the labor supply is fixed, competition among capital makes the real wage rate equal to labor productivity thereby bringing the exploitation rate and hence the profit rate to zero. Okishio's model does not explicitly formalize an unproductive sector and therefore it can be considered as replicating a system without unproductive sector. But the model has a labor market where the real wage rate is determined.

In comparison, our model demonstrates the same result without modeling the labor market. Because of that, however, i.e. because our model does not have a labor market to determine the wage rate, corollary 2 does not indicate that $\frac{\partial m}{\partial m_U} > 0$ and $\frac{\partial m}{\partial L_U} > 0$ will guarantee a positive profit. The reason is that as can be seen from the expression for Π_P^* in (29), whether $\Pi_P^* > 0$ will hold in that case depends on the specific values of the parameters including wage rate. That is, even when $\frac{\partial m}{\partial m_U} > 0$ and $\frac{\partial m}{\partial L_U} > 0$, $\Pi_P^* \leqslant 0$ is possible if w_P is too high.

Another side of the same story is that $\frac{\partial m}{\partial m_U} > 0$ and $\frac{\partial m}{\partial L_U} > 0$ can prevent the economy constrained by labor supply from experiencing zero profit as long as the wage rate is not too high. This way of understanding corollary 1 adds to the content of our characterization of the unproductive sector as socially necessary as described in definition 1. It suggests that in the economy constrained by labor supply the existence of unproductive sector makes it possible for the economy to avoid zero productive sector profits, which would be impossible without the unproductive sector.

[12] Remember from corollaries 1 and 2 that for both types of economy $m_P^* = m^*$ holds when there is no unproductive sector since in that situation $MVA_U^* = 0$ will be the case.

Moving on to our main issue on the relation of $m_P = m_U$, recall from section 4 that the relation was not an optimal condition for an economy with unconstrained labor supply. We can now verify from lemma 3 that $m_P^* \neq m_U^*$ and hence that $m_P = m_U$ is not an optimal condition for an economy constrained by labor supply, either. Then, as we did in section 4, let us extrapolate $m_P^* = m_U^*$ to investigate *ex post* implications of $m_P = m_U$.

First of all, under the condition of $\bar{L} = L_P + L_U$, equation (14) is rewritten as

$$\widetilde{m}^*(\widetilde{m}_U^*, \widetilde{L}_U^*, \alpha)(\bar{L} - \widetilde{L}_U^*) - \widetilde{m}_U^* \widetilde{L}_U^* - w_P(\bar{L} - \widetilde{L}_U^*) = (\widetilde{m}_U^* - w_P)(\bar{L} - \widetilde{L}_U^*) \tag{30}$$

Remember that tilde denotes the variable under the condition of $m_U^* = m_P^*$, and that the left–hand side is the value function of ideeller Gesamtkapitalist's optimization problem and the right–hand side is obtained from the definition of the optimal total profit of productive sectors, $m_P^* L_P - w_P L_P$ with $m_U^* = m_P^*$ extrapolated.

We obtain an interesting comparative statics result with respect to the parameter α.[13]

Proposition 5 *In an economy constrained by labor supply where $m_P^* = m_U^*$ holds, it follows that*

$$\frac{\partial \widetilde{L}_U^*}{\partial \alpha} \lesseqgtr 0 \Longleftrightarrow \frac{\partial m}{\partial \alpha} \gtreqless \frac{\partial \widetilde{m}_U^*}{\partial \alpha} \tag{31}$$

Proof. Taking the derivative of both sides of (30) w.r.t α yields $\frac{\partial m}{\partial \alpha}(\bar{L} - \widetilde{L}_U^*)$ for the left–hand side, where the envelope theorem is used, and $\frac{\partial \widetilde{m}_U^*}{\partial \alpha}(\bar{L} - \widetilde{L}_U^*) - \frac{\partial \widetilde{L}_U^*}{\partial \alpha}(\widetilde{m}_U^* - w_P)$ for the right–hand side. Dividing both sides by $\bar{L} - \widetilde{L}_U^*$ eventually gives

$$\frac{\partial m}{\partial \alpha} = \frac{\partial \widetilde{m}_U^*}{\partial \alpha} - \left(\frac{\widetilde{m}_U^* - w_P}{\bar{L} - \widetilde{L}_U^*}\right)\frac{\partial \widetilde{L}_U^*}{\partial \alpha} \tag{32}$$

Since $\bar{L} > L_U$ by definition and $m_U > w_P$ is most likely in any viable economy, $\left(\frac{\widetilde{m}_U^* - w_P}{\bar{L} - \widetilde{L}_U^*}\right) > 0$ holds and hence the relation in proposition 5 follows. ■

In comparison to the result $\frac{\partial m}{\partial \alpha} = \frac{\partial \widetilde{m}_U^*}{\partial \alpha}$ in proposition 1 for the economy with unconstrained labor supply, proposition 5 for the economy constrained by labor supply implies

[13] Concerning the comparative statics with respect to another parameter $\bar{L}$, which corresponds to proposition 2, we were not able to obtain any meaningful result.

$\frac{\partial m}{\partial \alpha} = \frac{\partial \widetilde{m}_U^*}{\partial \alpha}$ only when $\widetilde{m}_U^* = w_P$ and, otherwise, $\frac{\partial m}{\partial \alpha} \neq \frac{\partial \widetilde{m}_U^*}{\partial \alpha}$. Proposition 5 states that when α improves, the unproductive labor time will increases (decrease) if and only if the consequent increase in the value transfer to the unproductive sector is larger (smaller) than the consequent increase in the total value production.

5.2 Unproductive sector and capital accumulation

Similarly to section 4.2, we examine *ex post* implication of $m_U = m_P$ in relation to capital accumulation, by employing the concept of unproductive sector–led as suggested in definition 2. Since definition 2 requires the linear specification of m, we adopt (19) in this section as well. In that case, the optimal profit of productive sector in (29) is rewritten as

$$\Pi_P^* = \left[\frac{\beta_0 + \beta_3\alpha - w_P}{1+\beta_1} + \frac{\beta_1\beta_2}{(1+\beta_1)^2}\bar{L}\right]\bar{L} \tag{33}$$

Now, as a reference point, let us first examine the general case without $m_U = m_P$; the result is summarized as follows.

Proposition 6 *According to definition 2, the economy constrained by labor supply is unproductive sector–led when w_P is at least as high as a certain threshold since*

$$\frac{\partial \Pi_P^*}{\partial \beta_1} \gtreqless 0 \Longleftrightarrow w_P \gtreqless \beta_0 + \beta_3\alpha - \frac{\beta_2(1-\beta_1)}{1+\beta_1}\bar{L} \tag{34}$$

$$\frac{\partial \Pi_P^*}{\partial \beta_2} = \frac{\beta_1}{(1+\beta_1)^2}\bar{L}^2 > 0 \tag{35}$$

Proof. The results immediately follow from (33). ■

Proposition 6 states that when an economy constrained by labor supply is in a phase with too high a wage rate, it tends to rely, for accumulation and growth, on the unproductive sectors' socially necessary character of making the production of value and surplus value efficient.

The reason why the wage rate matters here especially in relation to the impact of β_1 as shown in (34) — in contrast to the results in propositions 3 and 4, where there is no such condition — is due to the existence of labor supply constraint. When the economy is constrained by labor supply, an expansion of the unproductive sector in terms of labor hour generates an effect that was absent in the case of an economy with unconstrained labor supply, i.e. a reduction in labor hours available for productive employment. In this case, too high a wage rate exerts an additional pressure on the profit.

This is the key to understanding the formal result of proposition 6. For the first condition (34) of proposition 6, consider an improvement in β_1 the qualitative effect of unproductive sector efficiency; from lemma 3 we can verify that it will lead to an increase in L_U^*, which was also true in the case of an economy unconstrained by labor supply — see lemma 1, remembering $\frac{\partial m}{\partial m_U} \equiv \beta_1$. When there is the labor supply constraint, the result of the increase in L_U^* is a reduction in $L_P^* \equiv \bar{L} - L_U^*$. Since the productive labor is the only genuine source of value production, the reduction in L_P^* lowers the money value added and essentially has a negative impact on the capital accumulation. But when the wage rate is so high as to significantly undermine the productive sector's profitability, the reduction in L_P^* makes the consequent saving of total wage cost sufficiently strong enough to outweigh its negative effect of lowering the money value added, thereby ultimately raising the profitability. This explains how the economy constrained by labor supply is unproductive sector–led when the wage rate is too high.

Our model treats the wage rate as constant and does not explain why it could be high or low. There could be various reasons for the wage variation. For one, the notable real wage growth during the post–war period was founded on strong trade union, welfare state policies, etc. The result was an erosion of capital profitability and the profit rate hitting the historic low in early 1980's, which brought about a capital flight to an alternative, easier source of profit, i.e. finance — which is the predominant section of unproductive sector. This line of profit squeeze explanation of the emergence of the so–called neoliberal regime dominated by financial sector is well explained by proposition 6.

Concerning the wage rate threshold $\beta_0 + \beta_3\alpha - \frac{\beta_2(1-\beta_1)}{1+\beta_1}\bar{L}$ in the first condition (34) of proposition 6, α and its coefficient β_3, among the other parameters, are noteworthy. If either of the two or both are sufficiently weak (strong) such that w_P is greater (smaller) than the threshold, the condition will (will not) be satisfied and the economy will (will not) be unproductive sector–led. That is, if the technological and market conditions of the productive sector are sufficiently unfavorable or their efficiency in contributing to value production is sufficiently weak, the economy can rely on the expansion of unproductive sector for accumulation and growth; otherwise, expanding the unproductive sector will rather dampen the productive sector profits.

Since the technology and market conditions are essential factors that affect labor productivity, the wage rate condition discussed above highlights that whether the wage rate is high or low relatively to the labor productivity is central in determining the economy's accumulation property in terms of productive vs. unproductive sector. That is, if the productive sector's wage rate is high relative to the labor productivity in that sector, the economy constrained by labor supply tends to be unproductive sector–led; otherwise, the economy will not be unproductive sector–led.

Let us now turn to our main question on the relation between capital accumulation and unproductive sector for the economy where $m_P = m_U$ is observed. For this, we impose $m_P^* = m_U^*$ on the optimal solutions listed in lemma 3. The consequently revised solutions are listed in lemma 5.

Lemma 5 *In an economy constrained by labor supply, the optimal solution under the condition of $m_P^* = m_U^*$:*

$$\widetilde{L}_U^* = \frac{\beta_1}{1+\beta_1}\bar{L} \tag{36}$$

$$\widetilde{m}_U^* = \frac{\beta_2\bar{L}}{(1+\beta_1)(2+\beta_1)} + \frac{w_P}{2+\beta_1} \tag{37}$$

$$\widetilde{m}_P^* = \frac{\beta_2\bar{L}}{(1+\beta_1)(2+\beta_1)} + \frac{w_P}{2+\beta_1} \tag{38}$$

$$\widetilde{m}^* = \frac{\beta_2\bar{L} + (1+\beta_1)w_P}{2+\beta_1} \tag{39}$$

$$\widetilde{\Pi}_P^* = \left(\frac{\beta_2\bar{L} - w_P}{2+\beta_1}\right)\bar{L} - \frac{\beta_1\beta_2}{(1+\beta_1)^2}\bar{L}^2 \tag{40}$$

Now we are ready to examine whether an economy constrained by labor supply exhibiting $m_P^* = m_U^*$ is unproductive sector–led. The result is summarized in proposition 7.

Proposition 7 *According to definition 2, the economy where $m_P^* = m_U^*$ is observed is unproductive sector–led in case w_P is at least as high as a certain threshold since*

$$\frac{\partial \widetilde{\Pi}_P^*}{\partial \beta_1} \gtreqless 0 \Longleftrightarrow w_P \gtreqless \beta_2 \bar{L}\Big[1 + \frac{(1-\beta_1)(2+\beta_1)^2}{(1+\beta_1)^3}\Big] \tag{41}$$

$$\frac{\partial \widetilde{\Pi}_P^*}{\partial \beta_2} = \frac{\bar{L}^2}{(2+\beta_1)(1+\beta_1)^2} > 0 \tag{42}$$

Proof. The results immediately follow from (40). ■

In exactly the same way as proposition 6, proposition 7 states that when the economy constrained by labor supply where $m_P = m_U$ holds suffers from too high a wage rate relative to a certain threshold, capital accumulation and growth can be achieved by relying on the unproductive sectors' socially necessary character of making the production of value and surplus value efficient. But there is one important difference, which has to do with the wage rate threshold. Remember that α and β_3 played an important role in interpreting the wage rate threshold in proposition 6 for the general case without $m_P^* = m_U^*$. By contrast, these parameters are absent in the wage rate threshold in the case of the economy where $m_P^* = m_U^*$ holds. Remembering that α captures the technological and market conditions of productive sector, this result implies that the conditions of the productive sector, which affect labor productivity in that sector, are inessential in determining whether the economy constrained by labor supply is unproductive sector–led or not.

Proposition 6, along with proposition 7, provides an important implication about the puzzling relation of $m_P = m_U$. The fact that $m_P = m_U$ has been observed in an economy constrained by labor supply implies that the economy can be either unproductive sector–led or not, depending on whether the wage rate is sufficiently high or low, but regardless of the labor productivity conditions of the productive sector. The observation of $m_P = m_U$ indicates that the impact the labor productivity conditions of the productive sector have on the economy's accumulation property in terms of productive vs. unproductive sectors, is neutralized.

6 Conclusion

$m_P = m_U$ is a puzzle from the perspective of Marxian transformation procedure. And that $m_P = m_U$ is observed in two countries quite consistently for around two to three decades — which are not short — is surprising. After posing a question whether the observation made in Korea and U.S. is an consequence of some equilibriating or optimizing mechanism, or simply a coincidence, we have derived a tentative answer that at least the co–movement of the two MELTs is not an optimal condition. Then, instead of further pursuing to find an explanation of the underlying mechanism, we took an indirect strategy of providing some *ex post* implications of the puzzling finding. They are summarized as follows.

First, in the economy with unconstrained labor supply where $m_P = m_U$ holds, when m rises (falls) due to an improvement (worsening) of the technology and market conditions of the productive sector, m_U also rises (falls) by the same degree. Since $m_P = m_U$, technology innovations and market condition improvement of the productive sector will make all of the MELTs, m, m_P, and m_U, increase by the exact same amount.

Second, in the economy with unconstrained labor supply where $m_P = m_U$ holds, the elasticity of m_U in response to a change in the productive labor time equals the ratio between unproductive and productive labor time; that is, if the unproductive labor time as a share of total labor time is larger, the increase in the productive labor time will lead to a greater value transfer to the unproductive sector.

Third, in the economy constrained by labor supply where $m_P = m_U$ holds, when the technology and market conditions of the productive sector improves, the unproductive labor time will increases (decrease) if and only if the consequent increase in the value transfer to the unproductive sector is larger (smaller) than the consequent increase in the total value production.

The fourth regards a growth implication of the unproductive sector. For the economy constrained by labor supply in general, when the wage rate in the productive sector is high relative to the conditions that affect the productive sector labor productivity, the economy can rely on the expansion of unproductive sector for capital accumulation and growth. In contrast, for the special case where $m_P = m_U$ holds, the labor productivity conditions in the productive sector become unimportant in deciding whether the productive sector wage rate is high or low and hence whether the economy is unproductive sector–led or not.

The paper can be developed in a couple of directions. First, we can enlarge the data to see if the puzzling result of $m_P = m_U$ holds in other countries. This practice may help get closer to gaining insights on the underlying mechanism of the puzzle. Second, we can estimate the efficiency coefficients, β_1, β_2, and β_3, with the help of time series regression to decide whether the wage rate is sufficiently high or low, which will allow us to tell whether the countries where $m_P = m_U$ is observed is unproductive sector–led or not.

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